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NOMOGRAMS FOR THE DOPPLER SHIFT VELOCIMETER

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ARO, Inc.

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ARNOLD ENGINEERING DEVELOPMENT CENTER

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NOTICES

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FOREWORD

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This technical report has been reviewed and is approved.

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ABSTRACT

The preliminary design of a laser Doppler velocimeter (LDV) for a particular application requires the determination of the frequency-to-velocity conversion constant and the spacial resolution of the focal volume. These parameters give the frequency range required of the signal-processing electronics and the size of the region in which velocity measurements are being made. These values are determined from the laser wavelength, the focal length of the focusing lens, the angle of the LDV beam pair intersection, and the diameter of the focused beams at the intersection. A set of nomographic scales has been developed to allow rapid and accurate determination of these parameters. They are also particularly useful in evaluating the effects of parameter variation and optimization of the LDV.

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SECTION I
INTRODUCTION

The preliminary design of a laser Doppler velocimeter (LDV) for a particular application requires the determination of the frequency-to-velocity conversion constant and the spacial resolution of the focal volume. These parameters give the frequency range required of the signal-processing electronics and the size of the region in which velocity measurements are being made. These values are determined from the laser wavelength, the focal length of the focusing lens, the angle of the LDV beam pair intersection, and the diameter of the focused beams at the intersection. A set of nomographic scales has been developed to allow rapid and accurate determination of these parameters. They are also particularly useful in evaluating the effects of parameter variation and optimization of the LDV.

SECTION II
FUNDAMENTAL EQUATIONS OF THE LDV

The frequency-to-velocity conversion constant for an LDV is determined by the relationship

$$f_D = \frac{2V}{\lambda_o} \sin(\theta/2) \tag{1}$$

where f_D is the Doppler-shifted frequency, V is the velocity being measured, λ_o is the laser wavelength, and θ is the angle formed by the two LDV beams making the measurement pair (Fig. 1). This relationship applies to both the reference beam (local oscillator) and dual-scatter LDV. Equation (1) is usually solved for V and is written as

$$V = \frac{f_D \lambda_o}{2 \sin(\theta/2)} \tag{2}$$

in units of fps.

The intersection of the two LDV beams forms a volume in space. This geometric volume is called the focal or probe volume and is the region in which velocity measurements are made.

This report considers the focal volume only as affected by the output optics. Additional work (Ref. 1) elaborates on the effect of the input or receiving optics on the focal volume. When the LDV input beams forming the focal volume have a Gaussian distribution, the volume is defined (Refs. 2 and 3) as having a surface intensity level of $1/e^2$ times

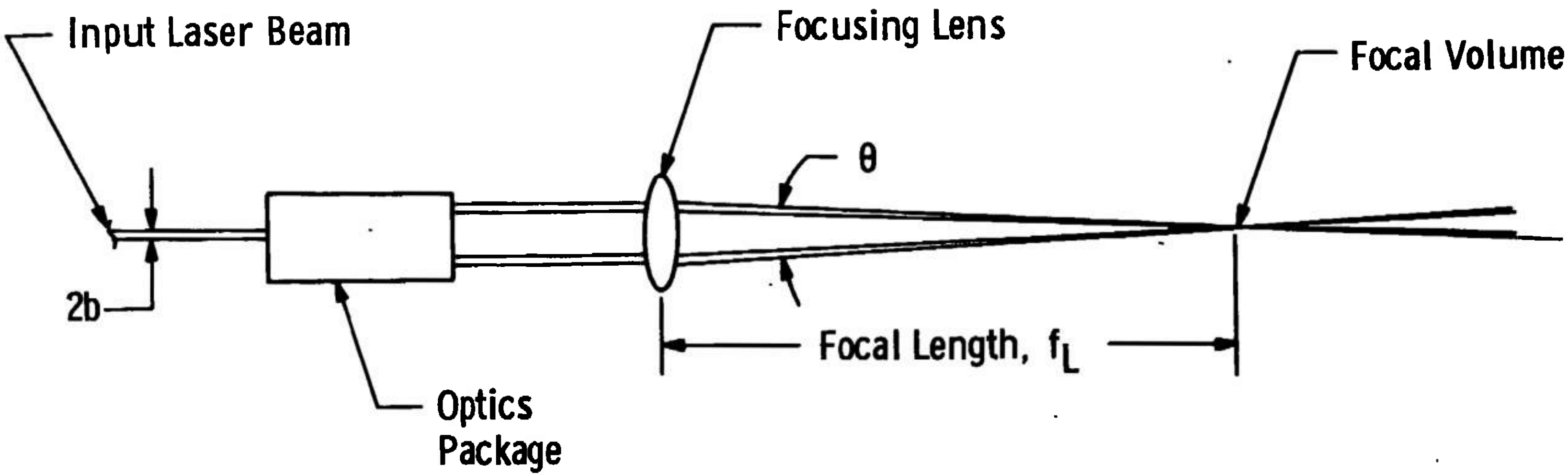

Fig. 1 Laser Doppler Velocimeter Arrangement

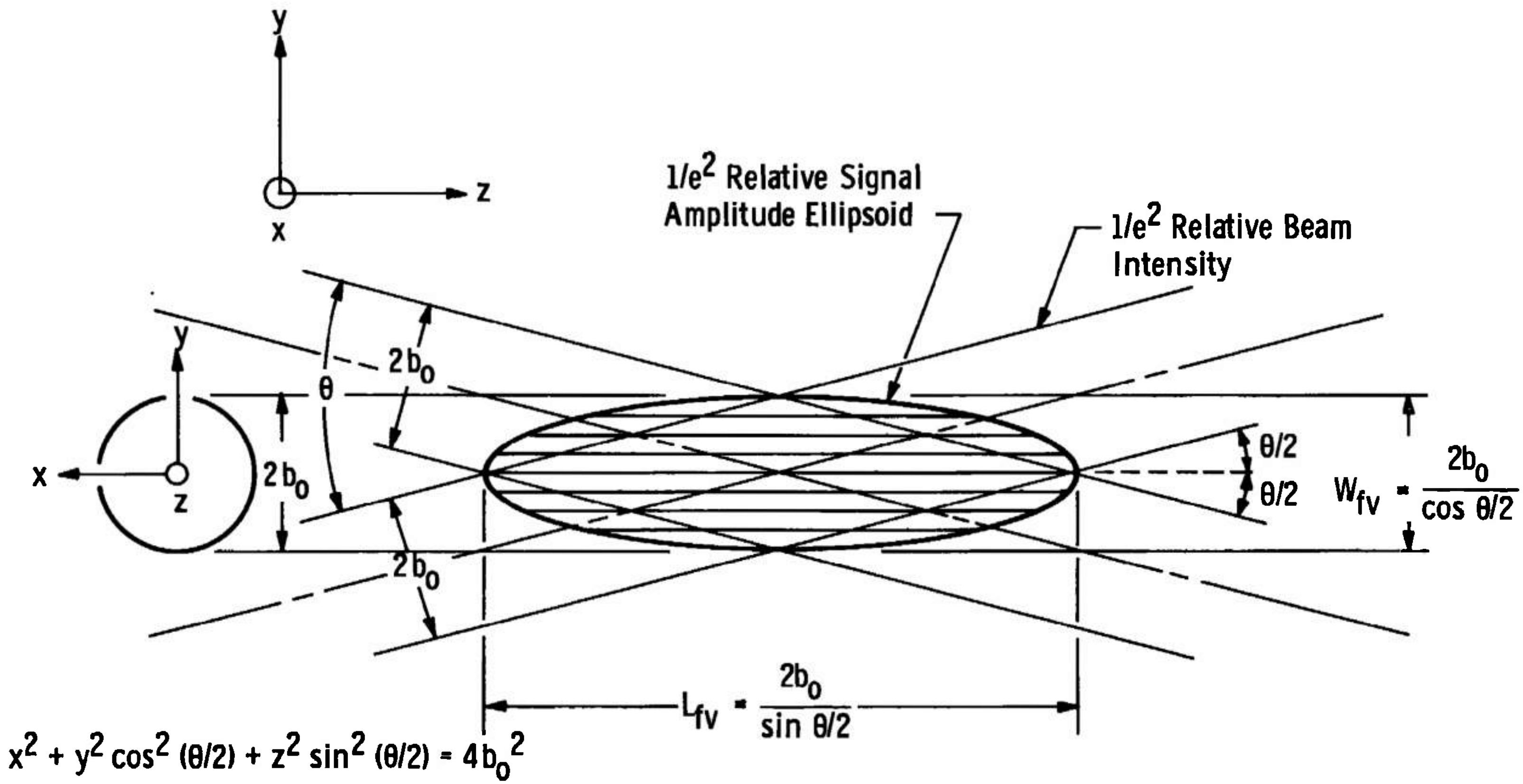

Fig. 2 Focal Volume Ellipsoid

the peak intensity of a single beam. By this definition the volume is an ellipsoid as in Fig. 2, whose equation is

$$x^2 + y^2 \cos^2(\theta/2) + z^2 \sin^2(\theta/2) = 4b_o^2 \quad (3)$$

The dimensions of interest of the volume are the length and maximum cross-sectional dimension. These define the spacial resolution of the focal volume. The length is, from Equation (3),

$$L_{fv} = 2\, b_o/\sin(\theta/2) \quad (4)$$

and the greatest cross-sectional dimension is

$$W_{fv} = 2\, b_o/\cos(\theta/2) \quad (5)$$

where b_o is the radius of the focused laser beams at their point of intersection in the focal volume. b_o is related to the laser output beam radius b, by

$$b_o = \frac{\lambda_o f_L}{\pi b} \quad (6)$$

SECTION III
FORMULATION OF THE NOMOGRAMS

There are four variables in the three equations to be solved. These are λ_o, b_o, θ, and either f_D or V. The equation for the velocity-frequency relationship,

$$V = \frac{f_D \lambda_o}{2 \sin(\theta/2)}$$

is divided into the product

$$V = \frac{1}{2 \sin(\theta/2)} (f_D \lambda_o)$$

and is solved using two interrelated nomograms. The terms λ_o and f_D are used to find their product $f_D \lambda_o$, which is then used in conjunction with θ to determine V. It is apparent that the value of any one parameter can be determined if the values of the other three are known.

The equation for the length of the focal volume

$$L_{fv} = 2\, b_o/\sin(\theta/2)$$

is divided into a product

$$L_{fv} = (2\, b_o) \left(\frac{1}{\sin(\theta/2)}\right)$$

and is solved using one nomogram.

The equation for the width of the focal volume,

$$W_{fv} = 2\,b_o/\cos\,(\theta/2)$$

is also divided and is solved in a similar fashion.

The equations for the length and width of the focal volume both have b_o, the radius of the focused laser beam appearing in them. The length and width of the focal volume cannot be determined, therefore, until a solution is made for b_o.

The equation for the focused beam radius,

$$b_o = \frac{f_L \lambda_o}{\pi b}$$

has been solved on a separate nomogram, Fig. 3. Before determining the length and width of the focal volume, b_o must be determined from Fig. 3.

SECTION IV
RANGE OF VARIABLES

The range of variables for use in the nomogram was restricted to those that are presently used for the measurement of fluid velocities. The wavelength of currently available lasers is between 3000 and 10,600Å. The scale was enlarged slightly to include the range from 1000 to 12,000Å. The angle θ currently in use varies between 5 and 10 deg. The θ scale was made 0 to 15 deg. These values of λ_o and θ when inserted into the velocity-frequency equation, give a range from 1.257 to 1131 fps/MHz. The frequency scale of the nomogram extends from 1 KHz to 1000 MHz. This allows the determination of velocities as low as 1.3×10^{-3} fps. Velocities much higher than 10^4 fps are presently uncommon in laser velocimetry, but the velocity scale is extended to 10^6 fps. The same range of θ, 0 to 15 deg, was used for the determination of focal volume length and width. The range of b_o is from 10^{-2} to 10 mm. The scale for the focal length of the lenses varies from 100 to 10,000 mm, which covers the range of usual applications.

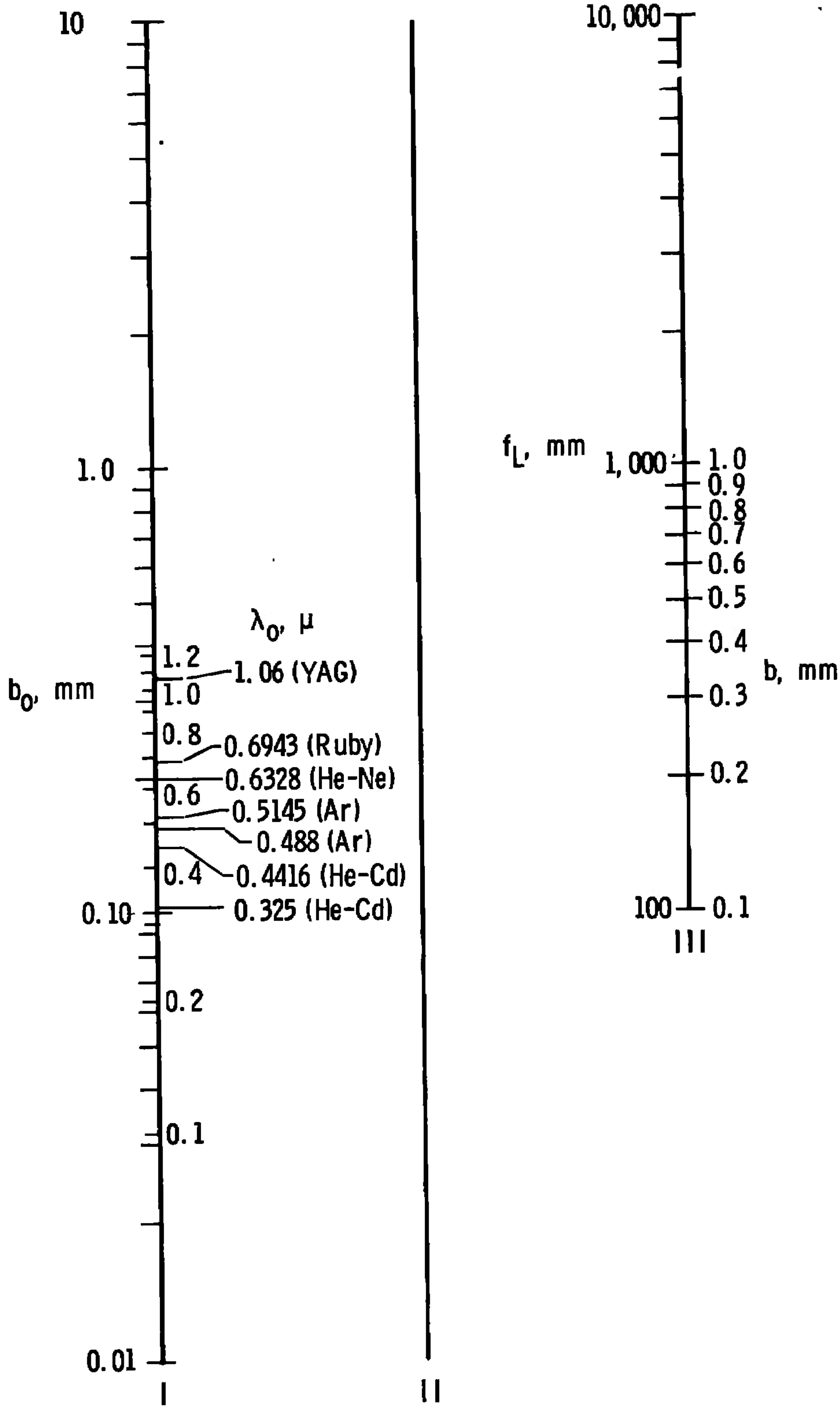

Fig. 3 Focused Beam Radius

SECTION V
COMBINATION NOMOGRAM

Since the three LDV equations, Eqs. (1), (4), and (5), all have the parameter θ in common, their separate nomograms may be combined into one nomogram for convenience (Ref. 4). This combined nomogram is shown in Fig. 4. Scales I through V are used in the velocity-frequency calculation, scales V through VII are used to calculate length of the focal volume, and scales V, VI, and VIII are used to calculate the width of the focal volume. The complete parametric determination for a given LDV can be made by three settings of a straight edge.

SECTION VI
EXAMPLE SOLUTION

The use of the nomogram may be better shown by an example. Given an LDV with an intersection angle θ of 10 deg to be used on an He-Ne laser ($\lambda_o = 0.6328\mu$), whose beam diameter is 1.0 mm (b = 0.5 mm), determine the velocity at an f_D of 10 MHz. Connecting 0.6328 on scale I with 10 MHz (10^7 Hz) on scale III gives an intersection on II. Rotating about this point to 10 deg on scale V gives an intersection on scale IV at a value of V approximately equal to 92 fps. If it was desired to know the Doppler-shifted frequency for this LDV at a velocity of 92 fps, it would be determined in the following way. The value of 92 fps on scale IV is connected with 10 deg on scale V, obtaining an intersection on scale II and then rotating about this intersection to connect with 0.6328 on scale I. The extension of this line gives an intersection on scale III of 10^7 Hz.

The determination of the focal volume length, L_{fv}, and the focal volume maximum width, W_{fv}, requires using b_o, the radius of the focused beams. The value of b_o is found from Fig. 3. Using the previous values and assuming a focusing lens of 400-mm focal length, b_o is found by connecting the 0.6328 point on column I with f_L = 400 mm on column III. This gives an intersection on column II about which the line is rotated to b - 0.5 mm on column III. The intersection on column I gives a b_o of 0.161 mm. With this value of b_o, the values of L_{fv} and W_{fv} are determined on Fig. 4. Connecting the 10-deg mark on column V with b_o = 0.161 on column VII gives L_{fv} = 3.7 mm on column VI and W_{fv} = 0.32 mm on column VIII. These scales can also be solved in other ways for particular variables.

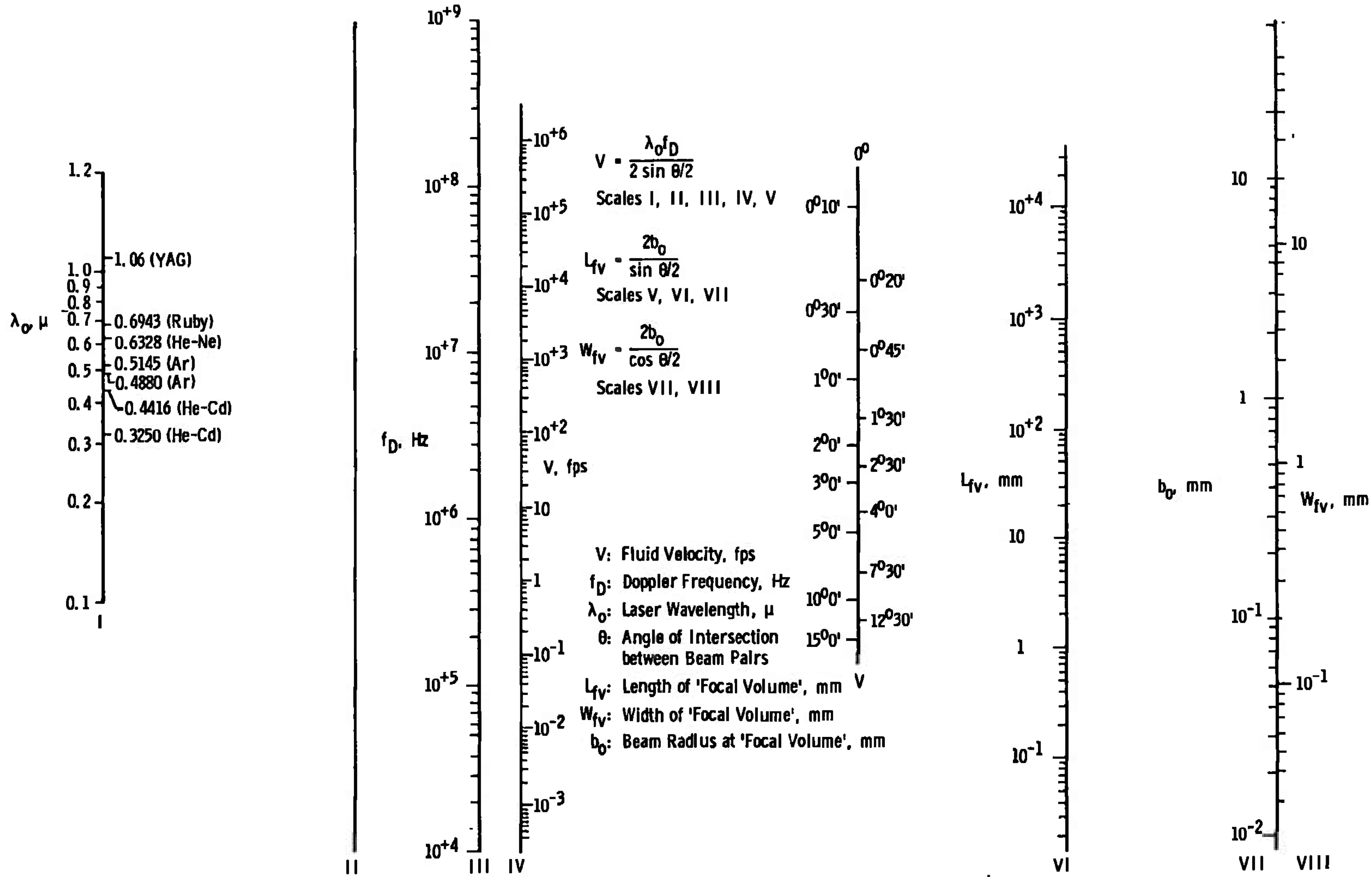

Fig. 4 Composite Nomogram

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